76/100

Exam I

Friday April 4, 2014

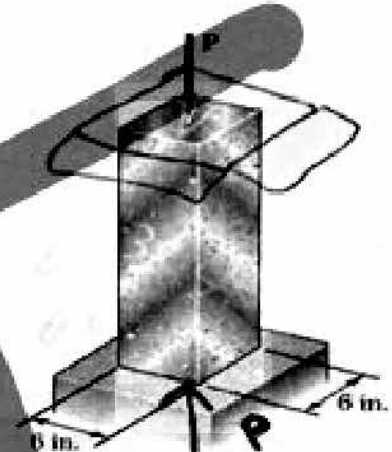
Duration: 1h 30min

All work must be shown to receive full credit.

Problem 1. (20 pts.) A centric load P is applied to the aluminum block shown below. The ultimate normal and shearing stresses of aluminum are $\sigma_u = 29$ ksi and $\tau_u = 7.25$ ksi. The modulus of elasticity is $E = 9.57 \times 10^6$ psi and poisson's ratio $\nu = 0.33$.

a - Calculate the maximum allowable load P that can be applied using a safety factor of 3 ? (12 pts.)

b - If the load in (a) is applied, calculate the edge length of the deformed square cross-sectional area which was originally 6 in. (8 pts.)



$\sigma_u = 29$ ksi

$\tau_u = 7,25$ ksi

$E = 9,57 \times 10^6$ psi

$\nu = 0,33$

$n = 3$

a) $\sigma_{all} = \frac{\sigma_u}{n} = \frac{29}{3} = 9,6667$ ksi

$\sigma_{all} = \frac{P}{A} \Rightarrow P = \sigma_{all} \times A$

$P = 9,6667 \times 10^3 \times (6 \times 6)$

$P = 348 \times 10^3$ lb (C)

$\tau_{all} = \frac{7,25 \times 10^3}{3} = 2416,6667$ psi

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$P = \tau_{all} \times A\ 2$

$= 2416,6667 \times 6 \times 6$

$= 87000,0012$ lb

$\boxed{P_{max} = 87000,0012 \text{ lb}}$

b)

$$\varepsilon_x = \frac{\sigma_x}{E}$$

$$\sigma_x = \frac{87000,0016}{6\times 6}$$

$$\sigma_x = 2416,667 \text{ psi}$$

$$\varepsilon_x = \frac{2416,667}{9,57\times10^{6}} = 2,525\times10^{-4}$$

(C)
~~52~~ moves down

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x}$$

$$\varepsilon_y = -\nu\,\varepsilon_x$$

$$\varepsilon_y = -0,33\times(-2,525\times10^{-4})$$

$$\varepsilon_y = 8,3325\times10^{-5}$$

$$\varepsilon_y = \frac{\Delta\text{edge}}{\text{edge}} \Rightarrow \Delta\text{edge} = 8,3325\times10^{-5}\times 6$$

$$= 4,9995\times10^{-4}$$

$$\approx 5\times10^{-4}\text{ in}$$

$$\boxed{\text{New edge} = 5\times10^{-4} + 6 = 6,0005 \text{ in}}$$

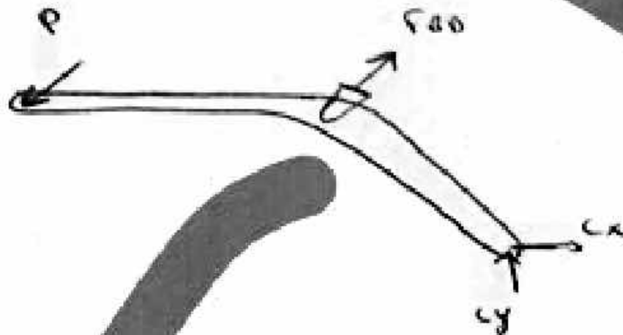
Problem 2. (20 pts.) Member ABC is supported by a steel pin and bracket (double shear) at C and a 0.5 m long steel cable BD with a 2 cm diameter. Assume that for steel, $E = 200$ GPa, $\tau_u = 100$ MPa and $\sigma_u = 200$ MPa. If the applied load $P = 20$ kN:

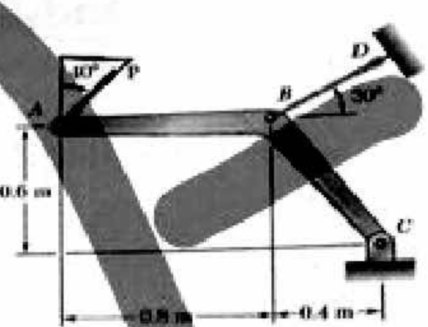
a - Calculate the normal stress in the cable. (10 pts.)

b - Calculate the elongation in the cable. (4 pts.)

c - Determine the minimum required pin diameter using a safety factor of 2. (6 pts.)

a)





$\circlearrowleft \Sigma M_C = 0$;

$$P \sin 40 \times 0{,}6 + P \cos 40 \times 1{,}2 - F_{BD} \cos 30 \times 0{,}6 - F_{BD} \sin 30 \times 0{,}4 = 0$$

$$P(\sin 40 \times 0{,}6 + \cos 40 \times 1{,}2) = F_{BD}(0{,}6 \cos 30 + 0{,}4 \sin 30)$$

$$P \times 1{,}305 = F_{BD} \times 0{,}72$$

$$F_{BD} = \frac{20 \times 10^3 \times 1{,}305}{0{,}72} = 36250 \text{ N.}$$

$$\sigma_{BD} = \frac{F_{BD}}{A_{BD}} = \frac{36250}{\frac{\pi}{4}(2 \times 10^{-2})^2} = \frac{36250}{3{,}1416 \times 10^{-4}} = 115{,}38 \text{ MPa}$$

b)

$$\delta_{BD} = \frac{P_{BD} L_{BD}}{A_{BD} E_{BD}} = \frac{36250 \times 0{,}5}{\frac{\pi}{4}(2 \times 10^{-2})^2 \times 200 \times 10^9}$$

$$= \frac{18125}{62831853{,}07} = 2{,}884683 \times 10^{-4} \text{ m}$$

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c) shear in pins

$$\tau_{all} = \frac{\tau_u}{2} = \frac{100}{2} = 50\,MPa$$

$\rightarrow \Sigma F_x = 0;$

$$-P\sin 40 + F_{00}\cos 30 - C_x = 0$$

$$C_x = F_{00}\cos 30 - P\sin 40$$

$$C_x = 36250 \times \cos 30 - 20000\sin 40$$

$$C_x = 18\,527{,}67\,N \ (\rightarrow)$$

$\uparrow \Sigma F_y = 0;$

$$-P\cos 40 + F_{00}\sin 30 + C_y = 0$$

$$-C_y = F_{00}\sin 30 - P\cos 40$$

$$C_y = P\cos 40 - F_{00}\sin 30$$

$$C_y = 20000\cos 40 - 36250 \times \sin 30$$

$$C_y = -2804{,}111138\,N$$ (C_y in opposite direction) ~~component~~ so downward

$$\tau_{all} = 50\,MPa = \frac{F_{CB}}{A}$$

$$F_{CB} = \sqrt{C_x^2 + C_y^2}$$

$$= \sqrt{3436452093 + 7863039{,}274}$$

$$= 18748{,}55723\,N$$

$$A = \frac{F_{CB}}{2 \times 50 \times 10^6} = \frac{18748{,}5536}{50 \times 10^6} = 3{,}75 \times 10^{-4}\,m^2$$

$$\frac{\pi}{4} d^2 = 3{,}75 \times 10^{-4}$$

$$\boxed{d_{min} > 0,02185\ m}$$

~~if the pin will fail it will be caused by the shear in BC.~~

$$\tau_{all} = \frac{F_{BC}}{2A_{pin\ bracket}} \Rightarrow A_{pin\ bracket} = \frac{F}{\tau} = \frac{18748,5532}{2\times 50\times 10^{6}}$$

$$= 1,8748\times 10^{-4}\ m^2$$

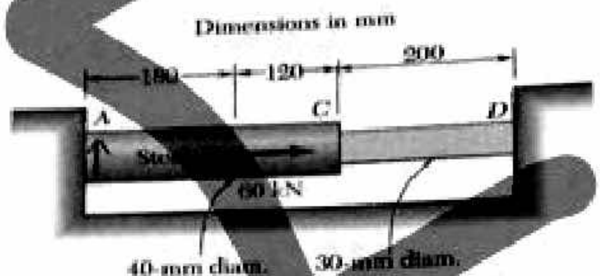
$$A = \frac{\pi}{4} d^2$$

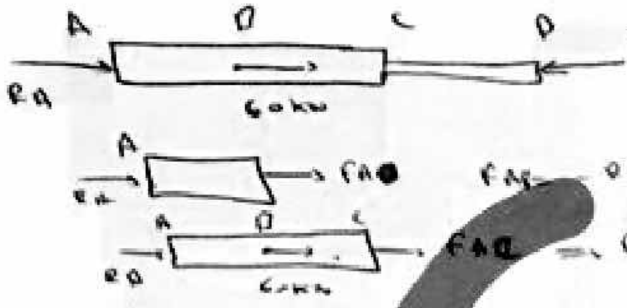
$$d = 0,01545\ m \Rightarrow \text{reject}$$

Problem 3. (20 pts.) Two cylindrical rods made of steel are joined at C and restrained by rigid supports at A and D. If for the steel used $E = 200$ GPa, calculate:

a - The maximum normal stress. (14 pts.)

b - The maximum shear stress. (6 pts.)





$\Rightarrow F_{BC} = -R_A - 60\times10^3$

$\rightarrow \Sigma F_x = 0 \Rightarrow \quad R_A + 60\times10^3 - R_E = 0$

$\delta_{AD} = 0$

$\delta_{AB} + \delta_{BC} + \delta_{CD} = 0$

$$\left(\frac{PL}{AE}\right)_{AB} + \left(\frac{PL}{AE}\right)_{BC} + \left(\frac{PL}{AE}\right)_{CD} = 0$$

$$\frac{-R_A \times 180\times10^{-3}}{\frac{\pi}{4}(40\times10^{-3})^2\times200\times10^9} + \frac{(-R_A - 60\times10^3)\times120\times10^{-3}}{\frac{\pi}{4}(40\times10^{-3})^2\times200\times10^9} + \frac{(-R_A - 60\times10^3)\times200\times10^{-3}}{\frac{\pi}{4}(30\times10^{-3})^2\times200\times10^9} = 0$$

$-112500\, R_A + 75000(-R_A - 60\times10^3) + (-R_A - 60\times10^3)\times 222222{,}222 = 0$

$-409722{,}222\, R_A - 4500000\times10^{3} - 13333333{,}33 = 0$

$\boxed{R_A = -43525{,}42372 \text{ N}}$ (opposite direction)

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$\Rightarrow R_E = R_A + 60\times10^{+3}$

$\boxed{R_E = 16474{,}57\text{N}}$

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a) $\sigma_B = \frac{F_B}{A_B} = \frac{\cancel{60\times10^3}}{\frac{\pi}{4}(40\times10^{-3})^2}$

$= 47,746 \text{ MPa}$

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b) $\sigma_A = \frac{43525,4372}{\frac{\pi}{4}(40\times10^{-3})^2} = 10,76 \text{ MPa}$

$\sigma_B = \frac{16474,52}{\frac{\pi}{4}(30\times10^{-3})^2} = 23,306 \text{ MPa}$

$\boxed{\sigma_{max} = 47,746 \text{ MPa}}$

5) $\tau_{AB} = \frac{43525,4372}{(40\times150)\times10^{-6}} = \frac{F}{A}$

$= 6045133,111 \text{ Pa}$

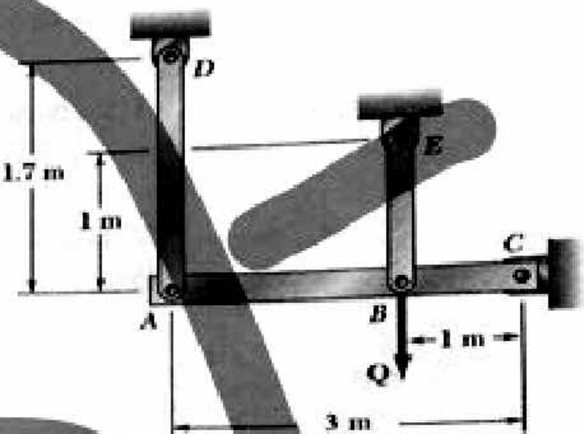
$\boxed{\tau_{B\,max} = \frac{60\times10^3}{(120\times40)\times10^{-6}} = 12500000 \text{ Pa}}$

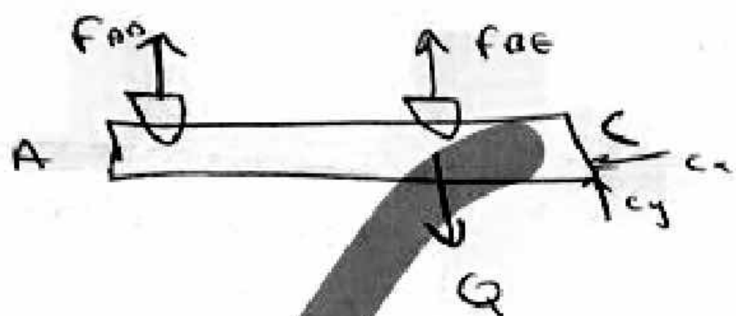
$\tau_B = \frac{16474,52}{(200\times30)\times10^{-6}} = 2745761,667 \text{ Pa}$

Problem 4. (20 pts.) The rigid bar ABC is supported by two links, AD and BE with 2.25×10^{-4} m^2 cross section and made of steel ($E = 200$ GPa). If $Q = 100$ kN, determine:

a - The normal stress in each of the links AD and BE. (14 pts.)

b - The maximum deflection of point B. (6 pts.)





$C_x = 0$

$\circlearrowleft \sum M_C = 0 ; - F_{AD} \times 3 - F_{BE} \times 1 + Q \times 1 = 0$

$3 F_{AD} + F_{BE} = 100 \times 10^3$

$\frac{\delta_B}{\delta_A} = \frac{1}{3} ; \delta_B = \frac{\delta_A}{3}$

$\delta_B = \frac{F_{BE} L_{BE}}{A_{BE} E_{BE}}$

$F_{BE} = \frac{\delta_A}{3} \times \frac{A_{BE} \times E_{BE}}{L_{BE}} = 15000000 \, \delta_A$

$\delta_A = \frac{F_{AD} \times L_{AD}}{A_{AD} \times E_{AD}} ; F_{AD} = \frac{\delta_A \times A_{AD} \times E_{AD}}{L_{AD}}$

$= 26470588.24 \, \delta_A$

$\Rightarrow 3 F_{AD} + F_{BE} = 100 \times 10^3$

$$79411764{,}72\,\delta_A + 150\,000\,000\,\delta_A = 100\times10^3$$

$$94411764{,}72\,\delta_A = 100\times10^3$$

$$\delta_A = 1{,}06\times10^{-3}\ m$$

$$\Rightarrow F_{AD} = 280\,568{,}23\ N \quad (T)$$

$$\Rightarrow F_{AE} = 15\,900\ N \quad (T)$$

$$\sigma_{AD} = \frac{F_{AD}}{A} = \frac{280568{,}23}{2{,}25\times10^{-4}} = \cancel{1247{,}05\ MPa} = 124{,}70511\ MPa$$

$$\sigma_{AE} = \frac{15900}{2{,}25\times10^{-4}} = 70{,}667\ MPa$$

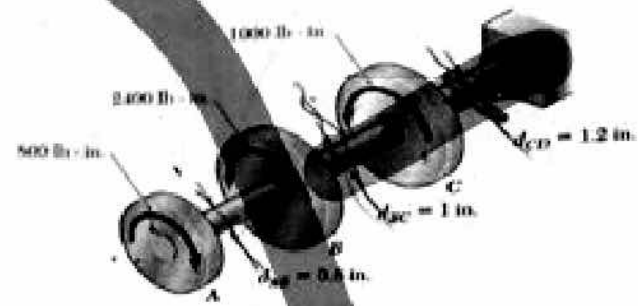
4)

$$\frac{\delta_D}{\delta_A} = \frac{1}{3}$$

$$\delta_D = \frac{\delta_A}{3} = \frac{1{,}06\times10^{-3}}{3} = 3{,}533\times10^{-4}\ m$$

Problem 6. (20 pts.) The length of shafts AB, BC, and CD are $L_{AB} = 2$ m, $L_{BC} = 2.5$ m and $L_{CD} = 4$ m, respectively. If the shafts are made of steel with $E = 29 \times 10^6$ psi and $G = 11.2 \times 10^6$ psi, determine:

- **a** - The maximum shear stress in the shaft system. (8 pts.)
- **b** - The angle of twist of B relative to C. (6 pts.)
- **c** - The angle of rotation at A. (6 pts.)



a)

$$\tau_A = \frac{Tc}{J}$$

$$J_{AB} = \frac{\pi}{2} c^4 = \frac{\pi}{2}\left(\frac{0.8}{2}\right)^4 = 0,040212 \text{ in}^4$$

$$J_{BC} = \frac{\pi}{2} c^4 = \frac{\pi}{2}(0,5)^4 = 0,09817477 \text{ in}^4$$

$$J_{CD} = 0,20357 \text{ in}^4$$

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$$\tau_A = \frac{800 \times 0,4}{0,040212} = 7957,82 \text{ psi}$$

$$\tau_B = \frac{(-2400+800)\times 0,5}{0,09817477} = -8148,733 \text{ psi}$$

$$\tau_C = \frac{(-2400-1000+800)\times 0,6}{0,20357} = -7663,2116 \text{ psi}$$

$$|\tau_{max}| = |\tau_B| = 8148,733 \text{ psi}$$

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b)

$$\theta_D = \frac{T_0 L_D}{J_D G_D}$$

$$= \frac{1000 \times 4 \times 2{,}07}{0{,}20337 \times 11{,}2 \times 10^6}$$

connect into in

$$= 3{,}645 \text{ rad}$$

$$\theta_C = \theta_D + \theta_C$$

$$= 3{,}645 + \frac{2400 \times 25 \times 2{,}07}{0{,}0381 7677 \times 11{,}2 \times 10^6}$$

$$= 3{,}645 + 0{,}01135$$

$$= 3{,}64035 \text{ rad}$$

b)

$$\theta_0 = \frac{T_0 L_D}{J_0 G_D}$$

$$= \frac{-1000 \times 4}{0{,}20337 \times 11{,}2 \times 10^6} = \left(-1{,}7544 \times 10^{-3} \text{ rad}\right)$$

$$\theta_C = \theta_0 + \theta_C$$

$$= -1{,}7544 \times 10^{-3} + \frac{2400 \times 25}{0{,}0381 7677 \times 11{,}2 \times 10^6}$$

$$= -1{,}7544 \times 10^{-3} + 5{,}6567 \times 10^{-3}$$

$$= 7{,}2111 \times 10^{-3} \text{ rad}$$

$$\phi_B = \frac{T_B L_B}{J_B G_B} = \frac{-2400 \times 2.5}{0.0577677 \times 16 \times 10^6}$$

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$$\boxed{\phi_B = -5.425 \times 10^{-3} \text{ rad}}$$ relative to C

considering $\phi_{B\,total} = \phi_B + \phi_C$

C+ $\phi_A = \phi_A + \phi_B + \phi_C$ $(\phi_C = \phi_D)$

$$\phi_A = \frac{T_A L_A}{J_A G_A} + (-5.425 \times 10^{-3}) + (-1.7544 \times 10^{-3})$$

$$\phi_A = \frac{800 \times 2}{0.0401126 \times 11.2 \times 10^6} - 5.425 \times 10^{-3} - 1.7544 \times 10^{-3}$$

$$= 3.5255 \times 10^{-3} - 5.425 \times 10^{-3} - 1.7544 \times 10^{-3}$$

$$= -3.6539 \times 10^{-3} \text{ rad}$$

(deformation in same direction as torques applied on B & C)

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